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Registration of hydrogen-like leptonic bound states ($e^-\mu^+$) and ($e^+\mu^-$) in reactions of high-energy scattering of polarized electrons and positrons by nuclei with Z $\sim$ 100 and analysis of CPT invariance

E.A. Choban and V.A. Ivanova[a]

Department of Nuclear Physics, State Polytechnical University of St. Petersburg, Polytechnicheskaya 29, 195251 St. Petersburg, Russian Federation

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Abstract. The cross-sections for the reactions of muonium (anti-muonium) production in the high-energy electron (positron) scattering by nuclei $e^-(e^+) + Z \to Z + M^0(\bar{M}^0) + \mu^-(\mu^+)$ are calculated in dependence on energy and polarization of the initial electron (positron) and polarization of the final $\mu^-(\mu^+)$-meson. Since this is a coherent phenomenon the cross-sections are proportional to Z^2. For $Z \sim 100$, due to the factor Z^2, the cross-sections are large enough to be measured at the energies available for the HERA Collider at DESY. The results are discussed in connection with a test of CPT invariance.

PACS. 11.10.-z Field theory – 13.40.-f Electromagnetic processes and properties – 13.66.De Lepton production in e^-e^+ interactions – 13.66.Jn Precision measurements in e^-e^+ interactions

1 Introduction

The standard model [1] represents the Lagrangian approach [2] to the description of strong, electromagnetic and weak interaction of elementary particles, based on the assumptions of locality and Lorentz invariance. Due to the Lüders-Pauli theorem (or the CPT theorem) [3] locality and Lorentz invariance of the Lagrangian of a quantum system lead to the invariance of a quantum system under CPT transformation which contains i) a charge conjugation (C), a replacement of all particles by their anti-particles, ii) a parity transformation (P), a reflection of spatial coordinates $(t, \vec{x}\,) \to (t, -\vec{x}\,)$, and iii) a time reversal (T), a reflection of time $(t, \vec{x}\,) \to (-t, \vec{x}\,)$. The simplest consequence of the CPT theorem is the equality of masses and lifetimes of particles and their anti-particles. At present these are the most experimentally well-verified requirements of the CPT theorem [4]. Nevertheless, theoretical and experimental tests for CPT invariance are still a well-motivated problem of elementary particle and nuclear physics [5]. This is related to the development of modern quantum field theories of strings and superstrings [6], which are more fundamental than the standard model and include it in the low-energy limit. Since string theories deal with extended non-local objects, the Lüders-Pauli theorem is not valid for these theories. A direct consequence of this can be a violation of CPT invariance for high-energy reactions of elementary particles and nuclei.

The problem of a test of CPT and Lorentz invariance has been recently discussed by Kostelecký with co-workers [7]. They suggested to check CPT and Lorentz invariance analysing the microwave spectroscopy of muonium M^0 [8,9]. Muonium M^0 is a leptonic hydrogen-like bound state of a positively charged muon μ^+ and an electron e^-. It was discovered in 1960 through the observation of its characteristic Larmor precession in a magnetic field [9]. The mean lifetime of muonium τ_{M^0} is approximately equal to the lifetime of a positively charged muon $\tau_{M^0} \simeq \tau_{\mu^+} = 2.197 \times 10^{-6}\,$s [1]. Due to the absence of strong interactions, muonium is an ideal system i) for determining the properties of muons, ii) for testing quantum electrodynamics [10], and iii) for searching for effects of unknown interactions in the electron-muon bound state [11]. Anti-muonium $\bar{M}^0$ is the leptonic analog of anti-hydrogen. It is a bound state of a negatively charged muon μ^- and a positron e^+.

A hydrogen-like structure of muonium allows to use atomic notations for the classification of its quantum states. For example, ${}^{2S+1}L_J$ corresponds to the quantum state of muonium (or anti-muonium) with a total angular

[a] e-mail: viola@kph.tuwien.ac.at

momentum (or a total spin) J, an angular momentum L and a spin S [1].

The use of muoniums M^0 and anti-muoniums $\bar{M}^0$ [1] as a laboratory for a test of CPT invariance has been recently suggested by Choban and Kazakov [12]. In their approach muoniums and anti-muoniums are produced with a total angular momentum $J = 0$ in the reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$ of high-energy scattering of electrons and positrons by nuclei with a number of protons Z. According to atomic classification, muonium (or anti-muonium) with a total angular momentum $J = 0$ can be in two bound states: i) a ground $1s$ state ${}^1\mathrm{S}_0$ with $L = S = 0$ and ii) an excited $2p$ state ${}^3\mathrm{P}_0$ with $L = S = 1$.

Due to the principle of superposition, muonium and anti-muonium should be produced in the reactions $e^- + Z \to Z+M^0+\mu^-$ and $e^++Z \to Z+\bar{M}^0+\mu^+$ in both states ${}^1\mathrm{S}_0$ and ${}^3\mathrm{P}_0$. The interference of these states should lead to time oscillations of the probability for the muonium (anti-muonium) to be detected at a moment t. A comparison of the time oscillations of the probabilities of the detected muonium and anti-muonium should testify whether CPT invariance is conserved or not. This is Choban-Kazakov's idea of a test of CPT invariance in the high-energy reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$. In terms of formulas it can be represented as follows.

Let the wave function of muonium produced in the reaction $e^- + Z \to Z + M^0 + \mu^-$ be defined by

$$\Psi_{M^0}(t, \vec{x}\,) = \sqrt{\frac{m_{M^0}}{|\vec{k}\,|}}\, e^{i\,\vec{k}\cdot\vec{x} - i\,Et}\,\Psi_{M^0}(t), \tag{1.1}$$

where E and $\vec{k}$ are the energy and 3-momentum of muonium, m_{M^0} is the mass of muonium. Note that the energy E does not contain the contributions of the binding energies E_{1s} and E_{2p} of the bound $1s$ and $2p$ states. The wave function $\Psi_{M^0}(t)$ can be written as

$$\Psi_{M^0}(t) = C_{1s}\,\exp\Big(-i\,\frac{m_{M^0}}{E}\,E_{1s}t\Big) + C_{2p}\,\exp\Big(-i\,\frac{m_{M^0}}{E}\,E_{2p}t\Big). \tag{1.2}$$

The coefficients C_{1s} and C_{2p} describe the contributions of the $1s$ and $2p$ states, respectively.

Introducing a parameter $\varepsilon = |C_{2p}|^2/|C_{1s} + C_{2p}|^2$, related to a fraction of the excited $2p$ state in the wave function of muonium $\Psi_{M^0}(t, \vec{x}\,)$ [12], the probability to find the muonium at moment t can be given by

$$P_{M^0}(t) = P_{M^0}(0)\,[1 - 4\sqrt{\varepsilon}\,(1 - \sqrt{\varepsilon})\sin^2(\Omega t)], \tag{1.3}$$

where $\Omega = m_\mu(E_{2p} - E_{1s})/2E = 5.103 \times 10^{-6} \times (m_\mu/E)\,\mathrm{MeV}$ [7][2].

It is seen that the probability $P_{M^0}(t)$ is an oscillating function. The period of oscillations T_{M^0} is determined by

$$T_{M^0} = \frac{2\pi}{\Omega} = \frac{4\pi}{E_{2p} - E_{1s}}\left(\frac{E}{m_\mu}\right) = 1.232 \times 10^6 \left(\frac{E}{m_\mu}\right)\mathrm{MeV}^{-1}. \tag{1.4}$$

In order to get T_{M^0} in seconds we have to multiply the r.h.s. of (1.4) by $\hbar = 6.582 \times 10^{-22}\,\mathrm{MeV\,s}$ [7]. This yields $T_{M^0} = 8.106 \times 10^{-16}\,(E/m_\mu)\,\mathrm{s}$. The period of oscillations T_{M^0} should be compared with the lifetime of muonium in the laboratory frame t_{M^0} which is related to the mean lifetime τ_{M^0} by the relativistic relation

$$t_{M^0} = \left(\frac{E}{m_\mu}\right)\tau_{M^0}\,. \tag{1.5}$$

Taking into account that $\tau_{M^0} \simeq 2.197 \times 10^{-6}\,\mathrm{s}$, we are able to estimate the number of oscillations ν_{M^0}:

$$\nu_{M^0} = \frac{t_{M^0}}{T_{M^0}} \simeq 2.710 \times 10^{9}. \tag{1.6}$$

An analogous expression can be written down for the probability $P_{\bar{M}^0}(t)$ to detect the anti-muonium at moment t with parameters $\bar{\varepsilon}$ and $\bar{\Omega}$. The result reads

$$P_{\bar{M}^0}(t) = P_{\bar{M}^0}(0)\,[1 - 4\sqrt{\bar{\varepsilon}}\,(1 - \sqrt{\bar{\varepsilon}})\sin^2(\bar{\Omega} t)]\,. \tag{1.7}$$

A relation of the probabilities $P_{M^0}(t)$ and $P_{\bar{M}^0}(t)$ to the experimental analysis of the violation of CPT invariance in the reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$ is the following.

For the calculation of the amplitude of muonium and anti-muonium production in the reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$ we use the effective Lagrangian of the $M^0\mu^+e^-$ interaction which can be defined as

$$\mathcal{L}_{M^0\mu^+e^-}(x) = g_{1s}\,\bar{\psi}_{\mu^-}(x)\gamma^5\psi_{e^-}(x)\,\Phi^\dagger_{1s}(x) + g_{2p}\,\bar{\psi}_{\mu^-}(x)\psi_{e^-}(x)\,\Phi^\dagger_{2p}(x)\,, \tag{1.8}$$

where $\bar{\psi}_{\mu^-}(x)$ and $\psi_{e^-}(x)$ are the local interpolating fields of the μ^+-meson and the electron e^-, $\Phi_{1s}(x)$ and $\Phi_{2p}(x)$ are the local operators of the interpolating fields of muonium in the states $1s$ and $2p$, respectively. They are expanded into plane waves and operators of creation and annihilation.

The wave functions of the relative motion of the muon μ^+ and the electron e^- contribute to the coupling constants g_{1s} and g_{2p}, which define the interaction of muonium in the $1s$ and $2p$ states with the μ^+e^- pair, respectively. For the calculation of the effective coupling constant we use the wave functions of muonium in the states ${}^1\mathrm{S}_0$ and ${}^3\mathrm{P}_0$ with the total momentum $\vec{P}$

[1] Anti-muonium $\bar{M}^0$ is a bound state of a negatively charged muon μ^- and a positron e^+. It is the leptonic analog of the anti-hydrogen.

[2] The account for a constant relative phase 2φ of coefficients C_{1s} and C_{2p} changes the probability (1.3) as follows: $P_{M^0}(t) = P_{M^0}(0)\,[1 - 4\sqrt{\varepsilon}\,(\sqrt{1-\varepsilon\,\sin^2\varphi} - \sqrt{\varepsilon}\,\cos\varphi)\,\sin(\Omega t + \varphi)\sin(\Omega t)]$.

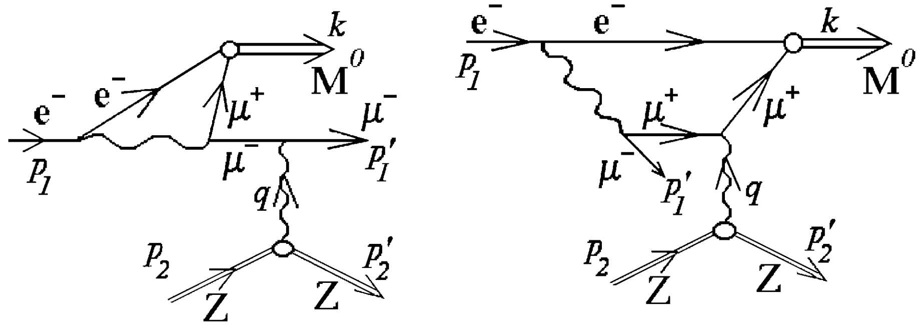

Fig. 1. Feynman diagrams of the amplitude of the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$.

defined by [13,14]

$$|M^0(\vec{P}\,);{}^1\mathrm{S}_0\rangle = \frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3k}{\sqrt{2E_{e^-}(\vec{k})}}\frac{\mathrm{d}^3q}{\sqrt{2E_{\mu^+}(\vec{q}\,)}}$$
$$\times\delta^{(3)}(\vec{P}-\vec{k}-\vec{q}\,)\,\varphi_{1s}(\vec{k})\,\frac{1}{\sqrt{2}}\,[b^\dagger_{e^-}(\vec{k},+1/2)d^\dagger_{\mu^+}(\vec{q},-1/2)$$
$$-b^\dagger_{e^-}(\vec{k},-1/2)d^\dagger_{\mu^+}(\vec{q},+1/2)]|0\rangle,$$
$$|M^0(\vec{P}\,);{}^3\mathrm{P}_0\rangle = \frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3k}{\sqrt{2E_{e^-}(\vec{k})}}\frac{\mathrm{d}^3q}{\sqrt{2E_{\mu^+}(\vec{q}\,)}}$$
$$\times\delta^{(3)}(\vec{P}-\vec{k}-\vec{q}\,)\,\varphi_{2p}(\vec{k})\frac{1}{\sqrt{2}}\,[b^\dagger_{e^-}(\vec{k},+1/2)d^\dagger_{\mu^+}(\vec{q},-1/2)$$
$$+b^\dagger_{e^-}(\vec{k},-1/2)d^\dagger_{\mu^+}(\vec{q},+1/2)]|0\rangle, \tag{1.9}$$

where $|0\rangle$ is the vacuum wave function; $b^\dagger_{e^-}(\vec{k},\sigma)$ $(b_{e^-}(\vec{k},\sigma)$ and $d^\dagger_{\mu^+}(\vec{k},\sigma)$ $(d_{\mu^+}(\vec{k},\sigma)$ are creation (annihilation) operators of the electron and muon μ^+ with momentum $\vec{k}$ and polarization $\sigma = \pm 1/2$. These operators obey the covariant canonical anti-commutation relations

$$\{b_{e^-}(\vec{k},\sigma), b^\dagger_{e^-}(\vec{k}\,',\sigma')\} = (2\pi)^3 2E_{e^-}(\vec{k})\,\delta^{(3)}(\vec{k}-\vec{k}\,')\delta_{\sigma\sigma\,'},$$
$$\{d_{\mu^+}(\vec{k},\sigma), d^\dagger_{\mu^+}(\vec{k}\,',\sigma')\} = (2\pi)^3 2E_{\mu^+}(\vec{k})\,\delta^{(3)}(\vec{k}-\vec{k}\,')\delta_{\sigma\sigma\,'}. \tag{1.10}$$

Then, $\varphi_{1s}(\vec{k})$ and $\varphi_{2p}(\vec{k})$ are the wave functions of the $1s$ and $2p$ states in the momentum representation. They are normalized to unity:

$$\int\frac{\mathrm{d}^3k}{(2\pi)^3}|\varphi_{1s}(\vec{k})|^2 = \int\frac{\mathrm{d}^3k}{(2\pi)^3}|\varphi_{2p}(\vec{k})|^2 = 1. \tag{1.11}$$

The wave functions (1.9) are normalized by

$$\langle{}^1\mathrm{S}_0;M^0(\vec{P}\,)|M^0(\vec{P}\,');{}^1\mathrm{S}_0\rangle = (2\pi)^3 2E^{(1s)}_{M^0}(\vec{P}\,)\,\delta^{(3)}(\vec{P}-\vec{P}\,'),$$
$$\langle{}^3\mathrm{P}_0;M^0(\vec{P}\,)|M^0(\vec{P}\,');{}^3\mathrm{P}_0\rangle = (2\pi)^3 2E^{(2p)}_{M^0}(\vec{P}\,)\,\delta^{(3)}(\vec{P}-\vec{P}\,'), \tag{1.12}$$

where $E^{(n)}_{M^0}(\vec{P}\,) = \sqrt{(m_{\mu^+}+m_{e^-}+E_n)^2+\vec{P}^{\,2}}$ is the total energy of muonium with $E_n = E_{1s}$ and $E_n = E_{2p}$ for the $1s$ and $2p$ states, respectively.

In the limit $m_e \to 0$ due to invariance of the interpolating electron field $\psi_{e^-}(x)$ under γ^5-transformation, $\psi_{e^-}(x) \to \gamma^5\psi_{e^-}(x)$, the effective Lagrangian (1.8) can be transcribed into the form

$$\mathcal{L}_{M^0\mu^+e^-}(x) = \bar{\psi}_{\mu^-}(x)\gamma^5\psi_{e^-}(x)\,(g_{1s}\,\Phi^\dagger_{1s}(x) + g_{2p}\,\Phi^\dagger_{2p}(x)). \tag{1.13}$$

Through the loop diagrams in fig. 1 the coupling constants g_{1s} and g_{2p} are related to the constants C_{1s} and C_{2p} (1.2).

Since one cannot distinguish experimentally the $1s$ and $2p$ states of muonium and of anti-muonium, the number of favourable events $N_{M^0}(T)$ and $N_{\bar{M}^0}(T)$, detected during an interval T, should be proportional to $\sigma^{(e^-Z)}_{M^0}(E_1)P_{M^0}(T)$ and $\sigma^{(e^+Z)}_{\bar{M}^0}(E_1)P_{\bar{M}^0}(T)$:

$$N_{M^0}(T) = \sigma^{(e^-Z)}_{M^0}(E_1)P_{M^0}(T)LT\,,$$
$$N_{\bar{M}^0}(T) = \sigma^{(e^+Z)}_{\bar{M}^0}(E_1)P_{\bar{M}^0}(T)LT\,, \tag{1.14}$$

where $\sigma^{(e^-Z)}_{M^0}(E_1)$ and $\sigma^{(e^+Z)}_{\bar{M}^0}(E_1)$ are the cross-sections for the reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$, respectively, E_1 is the energy of the initial electron and positron in the laboratory frame, and L is the luminosity of the collider.

Calculating the cross-sections in the CPT-invariant approximation, $\sigma^{(\vec{e}^{\,+}Z)}_{\bar{M}^0}(E_1) = \sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1)$, the ratio of the numbers of favourable events $R(T) = N_{M^0}(T)/N_{\bar{M}^0}(T)$ should be defined only by the ratio $P_{M^0}(T)/P_{\bar{M}^0}(T)$. It reads

$$R(T) = \frac{N_{M^0}(T)}{N_{\bar{M}^0}(T)} = \frac{P_{M^0}(T)}{P_{\bar{M}^0}(T)}. \tag{1.15}$$

Thus, measuring the ratio $R(T)$ of favourable events one can conclude that i) CPT invariance is broken if $R(T)$ depends on the time of observation and oscillates in time, and ii) CPT invariance is unbroken if $R(T)$ does not depend on the time of observation. Of course, this is only a qualitative test.

A practical realization of an experimental test of CPT invariance in high-energy reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$ depends on the statistics of favourable events $N = \sigma LT$ which can be detected during a certain interval of observation T. Nowadays, the HERA Collider at DESY operates 27.5 GeV electron and positron beams with luminosities

$L_{e^-} = (15\text{–}17) \times 10^{30}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} = (15\text{–}17)\,\mathrm{pb}^{-1}$ (H1–ZEUS) and $L_{e^+} = (65\text{–}68) \times 10^{30}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} = (65\text{–}68)\,\mathrm{pb}^{-1}$ (H1–ZEUS), respectively [15]. For these luminosities the number of events detected during one year for the production of muonium and anti-muonium are equal to $N_{M^0} = 500\,\sigma_{M^0}$ and $N_{\bar{M}^0} = 2100\,\sigma_{\bar{M}^0}$, where cross-sections σ_{M^0} and $\sigma_{\bar{M}^0}$ are measured in $1\,\mathrm{pb} = 10^{-36}\,\mathrm{cm}^2$.

Thus, the problem of the experimental realization of a test of CPT invariance suggested by Choban and Kazakov [12] is related to i) the values of the cross-sections for the reactions $e^- + Z \to Z + X^0 + \mu^-$ and $e^+ + Z \to Z + \bar{X}^0 + \mu^+$, defining the total number of favourable events and ii) a distinct signal that in the reactions $e^- + Z \to Z + X^0 + \mu^-$ and $e^+ + Z \to Z + \bar{X}^0 + \mu^+$, the states X^0 and $\bar{X}^0$ should be identified with muonium M^0 and anti-muonium $\bar{M}^0$, *i.e.* $X^0 = M^0$ and $\bar{X}^0 = \bar{M}^0$, respectively.

It is well known that a more detailed information about nuclear reactions can be obtained investigating polarizations of coupled particles. Therefore, in this paper we focus on the calculation of the cross-sections for the high-energy reactions $e^- + Z \to Z + M^0 + \mu^-$ and $e^+ + Z \to Z + \bar{M}^0 + \mu^+$ in dependence on the polarizations of the initial electron and positron and of the final muons μ^- and μ^+. Following [16], we denote these reactions as $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$. We suppose that the dependence on polarizations of final muons relative to polarizations of initial electrons and positrons should provide a necessary distinct signal confirming the production of muonium and anti-muonium with a total spin $J = 0$ in the reactions $\vec{e}^{\,-} + Z \to Z + X^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{X}^0 + \vec{\mu}^{\,+}$. Indeed, the processes competing with $\vec{e}^{\,-} + Z \to Z + X^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{X}^0 + \vec{\mu}^{\,+}$ are the reactions $\vec{e}^{\,\mp} + Z \to Z + \vec{e}^{\,\mp} + \mu^{\,+} + \mu^{\,-}$ of the production of the $\mu^+\mu^-$ pairs. In these reactions the momenta and polarizations of μ^+ and μ^- mesons are strongly correlated to each other and decorrelated with the polarization of the initial electron (or positron). Therefore, the detection of longitudinally polarized muons in the final state of the scattering of longitudinally polarized electrons (or positrons) by a nucleus Z should be a distinct signal for the production of muonium (or anti-muonium) with a total spin $J = 0$.

The paper is organized as follows. In sect. 2 we calculate the energy spectrum of the final muon and the cross-section for the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$. Since it is obvious that the CPT violation for the cross-sections is a negligible small effect which can be hardly measured, the cross-section is calculated assuming CPT invariance. This implies that the cross-section for the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ amounts to the cross-section for the reaction $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$, *i.e.* $\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1) = \sigma^{(\vec{e}^{\,+}Z)}_{\bar{M}^0}(E_1)$. In sect. 3 we estimate the contributions of the finite nuclear radius and the distortion of the wave functions of incoming and outcoming leptons caused by the strong Coulomb field induced by the electric charge Ze with $Z \sim 100$. We estimate that the contribution of the finite radius of the nucleus is of the order of a few percent. We show that the strong Coulomb field can hardly destroy the production of bound states of μ^+e^- and μ^-e^+ pairs, *i.e.* muoniums and anti-muoniums, in the reactions under consideration. This is by virtue of the time of the decays $M^0 \to \mu^+e^-$ and $\bar{M}^0 \to \mu^-e^+$ induced by the strong Coulomb field which is much greater than the time of the production of muonium and anti-muonium. In the conclusion we discuss the obtained results and a practical realization of experiments on the test of CPT invariance for the HERA Collider at DESY.

2 Cross-sections for reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$

Feynman diagrams describing the amplitude of the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ are depicted in fig. 1. The amplitude of the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ has been calculated in ref. [12] and reads

$$M(\vec{e}^{\,-}(p_1)Z(p_2) \to Z(p\,'_2)M^0(k)\vec{\mu}^{\,-}(p\,'_1)) = \frac{\alpha^2}{q^2}\frac{16\pi^2}{m_e}\frac{\Psi_{1s}(0)}{m_\mu^{3/2}}\frac{\ell^\mu L_\mu}{(q^2 - 2q\cdot k)}, \quad (2.1)$$

where ℓ^μ is the electromagnetic current of the nucleus and L_μ denotes the leptonic current

$$L_\mu = \bar{u}(p\,'_1, \sigma\,'_1)\,\gamma_5\,(\hat{q}\,p\,'_{1\mu} - q\cdot p\,'_1\,\gamma_\mu)u(p_1, \sigma_1), \quad (2.2)$$

where $u(p_1, \sigma_1)$ and $\bar{u}(p\,'_1, \sigma\,'_1)$ are the bispinorial wave functions of the initial electron and the final muon μ^-, $\Psi_{1s}(0) = 1/\sqrt{\pi a_{\mathrm{B}}^3}$ is the wave function of the muonium in the ground state, $a_{\mathrm{B}} = 1/m_e\alpha = 268.173\,\mathrm{MeV}^{-1}$ is the Bohr radius of muonium, $\alpha = 1/137.036$ is the fine-structure constant.

We would like to emphasize that the leptonic current L_μ is calculated in the ultra-relativistic limit, when masses of leptons are set zero. According to [12] this corresponds to the kinematical region, where the squared invariant mass of the pair $M^0\mu^-$, $\omega^2 = (p\,'_1 + k)^2$, is much greater than the squared mass of the μ^--meson m_μ^2, *i.e.* $\omega^2 \gg m_\mu^2$. In this kinematical region muonium with a total spin $J = 0$ behaves like a massless neutral scalar point-like particle.

The cross-section for the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ is defined by

$$\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1) = \frac{\alpha^7}{4\pi^2}\frac{m_e}{m_\mu^3}\frac{1}{m_Z E_1}\int\frac{T_{\mu\nu}R^{\mu\nu}}{q^4(p_1\cdot p\,'_1)^2}\,\delta^{(4)} \times(p\,'_2 + p\,'_1 + k - p_2 - p_1)\,\frac{\mathrm{d}^3k}{E}\frac{\mathrm{d}^3p\,'_1}{E\,'_1}\frac{\mathrm{d}^3p\,'_2}{E\,'_2}, \quad (2.3)$$

where E_1 is the energy of the initial electron in the laboratory frame coinciding with the rest frame of the target nucleus $p_{2\mu} = (m_Z, \vec{0}\,)$, then E, $E\,'_1$ and $E\,'_2$ are the energies of the muonium, the μ^--meson and the final nucleus, respectively. The tensors $R_{\mu\nu}$ and $T_{\mu\nu}$ are determined by

$$R_{\mu\nu} = \frac{1}{4}\,\mathrm{Sp}\{(\hat p_2 + m_Z)\ell^\dagger_\mu(\hat p\,'_2 + m_Z)\ell_\nu\} = F^2_{1Z}(q^2)\left[2\,p_{2\mu}p_{2\nu} - (p_{2\mu}q_\nu + p_{2\nu}q_\mu) + \frac{1}{2}\,q^2\,g_{\mu\nu}\right] + F^2_{2Z}(q^2)\left[2\,q^2\,m_Z^2\,g_{\mu\nu} + q^2\,(p_{2\mu}q_\nu + p_{2\nu}q_\mu) - q_\mu q_\nu\left(\frac{1}{2}\,q^2 + 2\,m_Z^2\right) - 2\,q^2\,p_{2\mu}p_{2\nu}\right] \tag{2.4}$$

and

$$T_{\mu\nu} = \frac{1}{4}\,\mathrm{Sp}\{(\hat p_1 - \gamma_5\hat w_1)L^\dagger_\mu(\hat p\,'_1 - \gamma_5\hat w\,'_1)L_\nu\} = \frac{1}{4}\,\mathrm{Sp}\{(\hat p_1 - \gamma_5\hat w_1)\gamma_5(q\cdot p\,'_1\,\gamma_\mu - \hat q\,p\,'_{1\mu}) \times(\hat p\,'_1 - \gamma_5\hat w\,'_1)\gamma_5(q\cdot p\,'_1\,\gamma_\nu - \hat q\,p\,'_{1\nu})\}, \tag{2.5}$$

where $F_{1Z}(q^2)$ and $F_{2Z}(q^2)$ are the form factors of a nucleus with a number of protons Z.

The polarization matrices $(\hat p_1 - \gamma_5\hat w_1)$ and $(\hat p\,'_1 - \gamma_5\hat w\,'_1)$ are obtained in the zero-mass limit from the standard polarization matrices $(\hat p_1 + m_e)(1-\gamma_5\hat a)$ and $(\hat p\,'_1 + m_\mu)\times(1-\gamma_5\hat b)$ [16], where a_μ and b_μ, the polarization 4-vectors of the initial electron and the final muon, are defined by

$$a_\mu = \left(\frac{\vec p_1\cdot\vec\xi_1}{m_e}, \vec\xi_1 + \frac{\vec p_1(\vec p_1\cdot\vec\xi_1)}{m_e(E_1+m_e)}\right),$$

$$b_\mu = \left(\frac{\vec p\,'_1\cdot\vec\xi\,'_1}{m_\mu}, \vec\xi\,'_1 + \frac{\vec p\,'_1(\vec p\,'_1\cdot\vec\xi\,'_1)}{m_\mu(E'_1+m_\mu)}\right). \tag{2.6}$$

The polarization 4-vectors a_μ and b_μ are normalized by $a_\mu a^\mu = a_0^2 - \vec a^{\,2} = -1$ and $b_\mu b^\mu = b_0^2 - \vec b^{\,2} = -1$. In turn, the polarization 3-vectors $\vec\xi_1$ and $\vec\xi\,'_1$ are normalized by $\vec\xi_1^{\,2} = \vec\xi\,'^{\,2}_1 = 1$. Recall that $p_1\cdot a = p\,'_1\cdot b = 0$.

According to definitions (2.6) the 4-vectors $w_{1\mu}$ and $w\,'_{1\mu}$ are equal to

$$w_{1\mu} = (\vec p_1\cdot\vec\xi_1, \vec n_1(\vec p_1\cdot\vec\xi_1)) = (\vec n_1\cdot\vec\xi_1)\,p_{1\mu},$$

$$w\,'_{1\mu} = (\vec p\,'_1\cdot\vec\xi\,'_1, \vec n\,'_1(\vec p\,'_1\cdot\vec\xi\,'_1)) = (\vec n\,'_1\cdot\vec\xi\,'_1)\,p\,'_{1\mu}, \tag{2.7}$$

where $\vec n_1 = \vec p_1/E_1$ and $\vec n\,'_1 = \vec p\,'_1/E'_1$ and $p_1\cdot w_1 = p\,'_1\cdot w\,'_1 = 0$ due to $p_1^2 = p\,'^{\,2}_1 = 0$. The analytical expression of $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = -\,2\,[1 + (\vec n_1\cdot\vec\xi_1)(\vec n\,'_1\cdot\vec\xi\,'_1)]\,(p_1\cdot p\,'_1)[(q\cdot p\,'_1)^2 g_{\mu\nu} - (q\cdot p\,'_1)(p\,'_{1\mu}q_\nu + p\,'_{1\nu}q_\mu) + q^2 p\,'_{1\mu}p\,'_{1\nu}]. \tag{2.8}$$

Due to the conservation of electric charge, the tensors $T_{\mu\nu}$ and $R_{\mu\nu}$ are gauge invariant

$$q^\mu T_{\mu\nu} = T_{\mu\nu}q^\nu = 0,$$
$$q^\mu R_{\mu\nu} = R_{\mu\nu}q^\nu = 0. \tag{2.9}$$

The cross-section for the reaction under consideration is then defined by

$$\sigma^{(\vec e^-Z)}_{M^0}(E_1) = \frac{\alpha^7}{\pi^2}\,\frac{m_e}{m_\mu^3}\,\frac{m_Z}{E_1}\int\frac{(-1)}{q^4(p_1\cdot p\,'_1)} \times[1 + (\vec n_1\cdot\vec\xi_1)(\vec n\,'_1\cdot\vec\xi\,'_1)]\Big\{(F^2_{1Z}(q^2) - q^2F^2_{2Z}(q^2))(q\cdot p\,'_1)^2 + \frac{q^2}{m_Z^2}\Big[(F^2_{1Z}(q^2) - q^2F^2_{2Z}(q^2))\Big((p_2\cdot p\,'_1)^2 - (q\cdot p\,'_1)(p_2\cdot p\,'_1)\Big) + \frac{1}{2}(F^2_{1Z}(q^2) + 4m_Z^2F^2_{2Z}(q^2)) \times(q\cdot p\,'_1)^2\Big]\Big\}\delta^{(4)}(p\,'_2 + p\,'_1 + k - p_2 - p_1)\,\frac{\mathrm{d}^3k}{E}\,\frac{\mathrm{d}^3p\,'_1}{E'_1}\,\frac{\mathrm{d}^3p\,'_2}{E'_2}. \tag{2.10}$$

The integration over the phase volume of the final state $ZM^0\mu^-$, we suggest to carry out in the non-relativistic limit of motion of the final nucleus [17]. In this approximation the 4-momentum of a final nucleus is equal to $p\,'_{2\mu} = (m_Z + \vec q^{\,2}/2m_Z, -\vec q) = (m_Z + T_2, -\vec q)$, then the transferred 4-momentum $q_\mu = (-T_2, \vec q)$ and $q^2 = -\vec q^{\,2}$.

In the non-relativistic limit of motion of the final nucleus the cross-section (2.10) reduces to the form

$$\sigma^{(\vec e^-Z)}_{M^0}(E_1) = Z^2\,\frac{\alpha^7}{\pi^2}\,\frac{m_e}{m_\mu^3}\,\frac{1}{E_1}\int\frac{1}{E_1E'_1 - \vec p_1\cdot\vec p\,'_1} \times\left[1 + (\vec n_1\cdot\vec\xi_1)\Big(\frac{\vec p\,'_1\cdot\vec\xi\,'_1}{E'_1}\Big)\right]\left(E_1'^{\,2} - \frac{(\vec q\cdot\vec p\,'_1)^2}{\vec q^{\,2}}\right) \times\delta(E'_1 + E + T_2 - E_1) \times\delta^{(3)}(\vec p\,'_1 + \vec k - \vec q - \vec p_1)\,\frac{\mathrm{d}^3k}{E}\,\frac{\mathrm{d}^3p\,'_1}{E'_1}\,\frac{\mathrm{d}^3q}{\vec q^{\,2}}, \tag{2.11}$$

where we have taken into account that $F_{1Z}(0) = Z$ [17] and that the main contribution comes from transferred momenta $\vec q^{\,2}$ co-measurable with zero. The former corresponds to the Weizsäcker-Williams approximation [18–23].

To simplify the calculation of the phase volume, we neglect the contribution of the kinetic energy of the final nucleus, which is small compared with the the typical transferred energies of coupled leptons. Integrating over $\vec k$, the 3-momentum of muonium, we get

$$\sigma^{(\vec e^-Z)}_{M^0}(E_1) = Z^2\,\frac{\alpha^7}{\pi^2}\,\frac{m_e}{m_\mu^3}\,\frac{1}{E_1}\int\frac{E'_1}{E_1E'_1 - \vec p_1\cdot\vec p\,'_1} \times\left[1 + (\vec n_1\cdot\vec\xi_1)\Big(\frac{\vec p\,'_1\cdot\vec\xi\,'_1}{E'_1}\Big)\right]\left(1 - \frac{(\vec q\cdot\vec p\,'_1)^2}{\vec q^{\,2}E_1'^{\,2}}\right) \times\delta(E_1 - E'_1 - |\vec p\,'_1 - \vec p_1 - \vec q|)\,\frac{\mathrm{d}^3p\,'_1}{|\vec p\,'_1 - \vec p_1 - \vec q|}\,\frac{\mathrm{d}^3q}{\vec q^{\,2}} = Z^2\,\frac{\alpha^7}{\pi^2}\,\frac{m_e}{m_\mu^3}\,\frac{1}{E_1}\int\left[1 + (\vec n_1\cdot\vec\xi_1)\Big(\frac{\vec p\,'_1\cdot\vec\xi\,'_1}{E'_1}\Big)\right] \times\frac{E'_1\,I(\vec p_1, \vec p\,'_1)}{E_1E'_1 - \vec p_1\cdot\vec p\,'_1}\,\mathrm{d}^3p\,'_1, \tag{2.12}$$

where we have put

$$I(\vec{p}_1,\vec{p}\,'_1)=\int\left(1-\frac{(\vec{q}\cdot\vec{p}\,'_1)^2}{\vec{q}\,^2E_1'^{\,2}}\right)\times\delta(E_1-E'_1-|\vec{p}\,'_1-\vec{p}_1-\vec{q}\,|)\,\frac{1}{|\vec{p}\,'_1-\vec{p}_1-\vec{q}\,|}\,\frac{\mathrm{d}^3q}{\vec{q}\,^2}. \tag{2.13}$$

The integration over $\vec{q}$ we carry out assuming that $|\vec{p}\,'_1-\vec{p}_1|\gg|\vec{q}\,|$ that is valid for the Weizsäcker-Williams approximation. Using the vector $\vec{z}=\vec{q}/|\vec{p}\,'_1-\vec{p}_1|$ we obtain

$$I(\vec{p}_1,\vec{p}\,'_1)=\int\left(1-\frac{(\vec{z}\cdot\vec{p}\,'_1)^2}{\vec{z}\,^2E_1'^{\,2}}\right)\times\delta\Big(E_1-E'_1-|\vec{p}\,'_1-\vec{p}_1|+(\vec{p}\,'_1-\vec{p}_1)\cdot\vec{z}\Big)\,\frac{\mathrm{d}^3z}{\vec{z}\,^2}. \tag{2.14}$$

Now it is convenient to introduce the new variables $x=E'_1/E_1$, $\vec{n}\,'_1=\vec{p}\,'_1/E'_1$ and $\vec{n}_1=\vec{p}_1/E_1$. In these variables the function $I(\vec{p}_1,\vec{p}\,'_1)$ reads

$$I(\vec{p}_1,\vec{p}\,'_1)=\frac{1}{E_1}\int\left(1-\frac{(\vec{z}\cdot\vec{n}\,'_1)^2}{\vec{z}\,^2}\right)\times\delta\Big(1-x-|x\vec{n}\,'_1-\vec{n}_1|+(x\vec{n}\,'_1-\vec{n}_1)\cdot\vec{z}\Big)\,\frac{\mathrm{d}^3z}{\vec{z}\,^2}. \tag{2.15}$$

The next step in the integration over $\vec{z}$ is to rewrite the integral in the following form:

$$I(\vec{p}_1,\vec{p}\,'_1)=\frac{1}{\pi E_1}\mathcal{R}e\int_0^\infty\mathrm{d}\lambda\,e^{i\lambda(1-x-|x\vec{n}\,'_1-\vec{n}_1|)}\times\int\left(1+\frac{1}{\lambda^2\vec{z}\,^2}\frac{\partial^2}{\partial x^2}\right)e^{i\lambda(x\vec{n}\,'_1-\vec{n}_1)\cdot\vec{z}}\,\frac{\mathrm{d}^3z}{\vec{z}\,^2}. \tag{2.16}$$

Since the integrals over $\vec{z}$ are equal to

$$\int e^{i\lambda(x\vec{n}\,'_1-\vec{n}_1)\cdot\vec{z}}\,\frac{\mathrm{d}^3z}{\vec{z}\,^2}=\frac{4\pi}{\lambda|x\vec{n}\,'_1-\vec{n}_1|}\int_0^\infty\frac{\sin z}{z}\,\mathrm{d}z=$$
$$\frac{4\pi}{\lambda|x\vec{n}\,'_1-\vec{n}_1|}\lim_{\alpha\to1}\int_0^\infty\frac{\sin z}{z^\alpha}\,\mathrm{d}z=$$
$$\frac{4\pi}{\lambda|x\vec{n}\,'_1-\vec{n}_1|}\lim_{\alpha\to1}\mathcal{I}m\int_0^\infty\mathrm{d}z\,e^{iz}z^{-\alpha}=$$
$$\frac{4\pi}{\lambda|x\vec{n}\,'_1-\vec{n}_1|}\lim_{\alpha\to1}\mathcal{I}m\,\frac{\Gamma(1-\alpha)}{(-i)^{1-\alpha}}=$$
$$\frac{4\pi}{\lambda|x\vec{n}\,'_1-\vec{n}_1|}\lim_{\alpha\to1}\Gamma(2-\alpha)\,\frac{\sin\left(\frac{\pi}{2}(1-\alpha)\right)}{1-\alpha}=\frac{2\pi^2}{\lambda|x\vec{n}\,'_1-\vec{n}_1|} \tag{2.17}$$

and

$$\int e^{i\lambda(x\vec{n}\,'_1-\vec{n}_1)\cdot\vec{z}}\,\frac{\mathrm{d}^3z}{\vec{z}\,^4}=4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\int_0^\infty\frac{\sin z}{z^3}\mathrm{d}z=$$
$$4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\lim_{\alpha\to3}\mathcal{I}m\int_0^\infty\mathrm{d}z\,e^{iz}z^{-\alpha}=$$
$$4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\lim_{\alpha\to3}\mathcal{I}m\,\frac{\Gamma(1-\alpha)}{(-i)^{1-\alpha}}=$$
$$4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\lim_{\alpha\to3}\Gamma(1-\alpha)\sin\left(\frac{\pi}{2}(1-\alpha)\right)=$$
$$4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\lim_{\alpha\to3}\Gamma(4-\alpha)\frac{\sin\left(\frac{\pi}{2}(1-\alpha)\right)}{(1-\alpha)(2-\alpha)(3-\alpha)}=$$
$$-4\pi\lambda|x\vec{n}\,'_1-\vec{n}_1|\lim_{\alpha\to3}\Gamma(4-\alpha)\times\frac{\sin\left(\frac{\pi}{2}(3-\alpha)\right)}{(1-\alpha)(2-\alpha)(3-\alpha)}=-\pi^2\lambda|x\vec{n}\,'_1-\vec{n}_1|, \tag{2.18}$$

the function $I(\vec{p}_1,\vec{p}\,'_1)$ is defined by the integral over λ

$$I(\vec{p}_1,\vec{p}\,'_1)=\frac{\pi}{E_1}\left(\frac{1}{|x\vec{n}\,'_1-\vec{n}_1|}+\frac{(x-\vec{n}\,'_1\cdot\vec{n}_1)^2}{|x\vec{n}\,'_1-\vec{n}_1|^3}\right)\times\int_0^\infty\frac{\mathrm{d}\lambda}{\lambda}\,\cos(\lambda(1-x-|x\vec{n}\,'_1-\vec{n}_1|)). \tag{2.19}$$

The integral over λ is divergent. However, it can be regularized by following the theory of generalized functions [24]. The result reads

$$I(\vec{p}_1,\vec{p}\,'_1)=\frac{\pi}{E_1}\left(\frac{1}{|x\vec{n}\,'_1-\vec{n}_1|}+\frac{(x-\vec{n}\,'_1\cdot\vec{n}_1)^2}{|x\vec{n}\,'_1-\vec{n}_1|^3}\right)\times\ln\left(\frac{1}{|x\vec{n}\,'_1-\vec{n}_1|-(1-x)}\right). \tag{2.20}$$

Substituting (2.20) in (2.12) and proceeding to variables x and $\vec{n}\,'_1$ we define the energy spectrum of the final μ^--meson:

$$\frac{1}{x^2}\frac{\mathrm{d}\sigma^{(\tilde{e}^-Z)}_{M^0}(E_1)}{\mathrm{d}x}=Z^2\,\frac{\alpha^7}{\pi}\,\frac{m_e}{m_\mu^3}\int\frac{1+(\vec{n}_1\cdot\vec{\xi}_1)(\vec{n}\,'_1\cdot\vec{\xi}\,'_1)}{1-\vec{n}_1\cdot\vec{n}\,'_1}\times\left(\frac{1}{|x\vec{n}\,'_1-\vec{n}_1|}+\frac{(x-\vec{n}\,'_1\cdot\vec{n}_1)^2}{|x\vec{n}\,'_1-\vec{n}_1|^3}\right)\times\ln\left(\frac{1}{|x\vec{n}\,'_1-\vec{n}_1|-(1-x)}\right)\mathrm{d}\Omega_{\vec{n}\,'_1}. \tag{2.21}$$

For the subsequent integration over the unit vector $\vec{n}\,'_1$ we introduce the angular variables as follows:

$$\vec{n}_1\cdot\vec{n}\,'_1=\cos\vartheta'_1,$$
$$\vec{n}\,'_1\cdot\vec{\xi}\,'_1=\cos\vartheta'_1\cos\Theta'_1+\sin\vartheta'_1\sin\Theta'_1\cos(\varphi'_1-\Phi'_1),$$
$$\mathrm{d}\Omega_{\vec{n}\,'_1}=\sin\vartheta'_1\mathrm{d}\vartheta'_1\mathrm{d}\varphi'_1, \tag{2.22}$$

where Θ_1' and Φ_1' are polar and azimuthal angles of the polarization vector $\vec{\xi}\,'_1$ relative to the momentum $\vec{p}_1$. In (2.22) we have taken into account that $|\vec{\xi}\,'_1| = 1$. Integrating over φ_1' we get

$$\frac{1}{x^2}\frac{\mathrm{d}\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1)}{\mathrm{d}x} = 2Z^2\alpha^7\frac{m_e}{m_\mu^3}\int\limits_0^\pi \frac{1+(\vec{n}_1\cdot\vec{\xi}_1)\cos\vartheta_1'\cos\Theta_1'}{1-\cos\vartheta_1'}$$
$$\times\left(\frac{1}{\sqrt{1-2x\cos\vartheta_1'+x^2}}+\frac{(x-\cos\vartheta_1')^2}{(1-2x\cos\vartheta_1'+x^2)^{3/2}}\right)$$
$$\times\ln\left(\frac{1}{\sqrt{1-2x\cos\vartheta_1'+x^2}-(1-x)}\right)\sin\vartheta_1'\mathrm{d}\vartheta_1'\,. \quad (2.23)$$

Now it is convenient to introduce a new variable $t = \sqrt{1-2x\cos\vartheta_1'+x^2}$, which varies in the limits $1-x \le t \le 1+x$. In terms of t the energy spectrum (2.23) reads

$$\frac{1}{x}\frac{\mathrm{d}\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1)}{\mathrm{d}x} =$$
$$2Z^2\alpha^7\frac{m_e}{m_\mu^3}\int\limits_{1-x}^{1+x}\frac{2x+(1+x^2-t^2)(\vec{n}_1\cdot\vec{\xi}_1)\cos\Theta_1'}{t^2-(1-x)^2}$$
$$\times\Big(1+\frac{(1-x^2-t^2)^2}{4x^2t^2}\Big)\,\ell n\Big(\frac{1}{t-(1-x)}\Big)\,\mathrm{d}t. \quad (2.24)$$

It is seen that the integral over t is concentrated in the vicinity of the lower limit. The singularity of the integrand in the vicinity of the lower limit can be easily regularized by making a change of the lower limit $1-x \to 1-x+\Lambda^2/E_1^2$, where Λ is a cut-off restricting energies of the final μ^--meson from below. According to the kinematical region $\omega^2 \gg m_\mu^2$ [12] the cut-off Λ can be chosen of order of $\Lambda \simeq 1\,\mathrm{GeV}$. Such a dependence on the cut-off Λ can be justified as follows: $E\,'_1 = |\vec{p}\,'_1| = \sqrt{(\vec{p}\,'_1)^2+\Lambda^2-\Lambda^2} = \sqrt{(\vec{p}\,'_1)^2+\Lambda^2} - \Lambda^2/\sqrt{(\vec{p}\,'_1)^2+\Lambda^2} \to E\,'_1 - \Lambda^2/E\,'_1 \approx E\,'_1 - \Lambda^2/E_1$.

Keeping only the dominant contributions to the integral over t, we get

$$\frac{\mathrm{d}\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1)}{\mathrm{d}x} = 8Z^2\alpha^7\frac{m_e}{m_\mu^3}\ln^2\left(\frac{E_1}{\Lambda}\right)$$
$$\times\frac{x^2}{1-x}\,[1+(\vec{n}_1\cdot\vec{\xi}_1)\cos\Theta_1'\,]. \quad (2.25)$$

Introducing the angle Θ_1, defined by $\vec{n}_1\cdot\vec{\xi}_1 = \cos\Theta_1$, where we have taken into account that $|\vec{\xi}_1| = 1$, we obtain the energy spectrum of μ^--mesons for the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ in dependence on the polarizations of the initial electron and the final μ^--muon described by the angles Θ_1 and $\Theta\,'_1$:

$$\frac{\mathrm{d}\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1)}{\mathrm{d}x} = 8Z^2\alpha^7\frac{m_e}{m_\mu^3}\ln^2\left(\frac{E_1}{\Lambda}\right)$$
$$\times\frac{x^2}{1-x}\,(1+\cos\Theta_1\cos\Theta_1'\,). \quad (2.26)$$

Integrating over x, we arrive at the total cross-section for the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$:

$$\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1) = 16Z^2\alpha^7\frac{m_e}{m_\mu^3}\ln^3\left(\frac{E_1}{\Lambda}\right)(1+\cos\Theta_1\cos\Theta_1'\,). \quad (2.27)$$

Assuming that electrons are longitudinally polarized electrons, $\cos\Theta_1 = 1$, one can see that for fixed electron energy the cross-section acquires the maximal value only for longitudinally polarized muons, $\cos\Theta_1' = 1$. This agrees with the production of muonium with a total spin $J = 0$. Thus, we argue that the appearance of longitudinally polarized muons in the final state of the reaction $\vec{e}^{\,-} + Z \to Z + X + \vec{\mu}^{\,-}$ should testify the production of muonium $X \equiv M^0$.

For the numerical estimate of the cross-sections at the energies available for the HERA Collider at DESY [15], *i.e.* $E_1 = 27.5\,\mathrm{GeV}$, we suggest to use radon, $^{222}_{86}\mathrm{Rn}$, as a target nucleus, since radon has spin 1/2. The cross-sections for longitudinally polarized electrons and positrons scattering by $^{222}_{86}\mathrm{Rn}$ and longitudinally polarized muons are equal to

$$\sigma^{(\vec{e}^{\,-}\mathrm{Rn})}_{M^0}(E_1 = 27.5\,\mathrm{GeV}) =$$
$$\sigma^{(\vec{e}^{\,+}\mathrm{Rn})}_{\bar{M}^0}(E_1 = 27.5\,\mathrm{GeV}) = 1.6\,\mathrm{pb}. \quad (2.28)$$

In our calculation the cross-section for the reaction $e^- + Z \to Z + M^0 + \mu^-$ has turned out to be dependent on the cut-off $\Lambda \simeq 1\,\mathrm{GeV}$. In this connection we would like to remind that the problem of the appearance of a cut-off in the cross-sections for some reactions calculated within the Weizsäcker-Williams approximation has been pointed out by Bertulani and Baur [20].

Now let us discuss the energy dependence of the cross-section (2.27). It is well known that for the e^+e^- pair production in heavy-ion collisions [20–22] and $p\bar{p}$ collisions [23] the cross-section for a capture of the final electron in an atomic K-shell orbit is proportional to $\ln(\gamma_{\mathrm{coll}})$, where γ_{coll} is a Lorentz factor of colliding particles in the center-of-mass frame. This factor is related to the corresponding Lorentz factor γ_{p} of the projectile (for a fixed-target machine) by $\gamma_{\mathrm{p}} = 2\gamma^2_{\mathrm{coll}} - 1$ [20,22], where $\gamma_{\mathrm{p}} \sim E_1$.

In turn, the cross-section for the production of a point-like neutral scalar particle in high-energy heavy-ion collisions in the Weizsäcker-Williams approximation is proportional to $\ln^3(\gamma_{\mathrm{coll}})$ [20,22].

For very high energies, when masses of coupled leptons can be neglected, muonium with a total spin $J = 0$ can be treated as a point-like massless scalar neutral particle. Such a property of the muonium is caused by an additional pole singularity appearing at $(q-k)^2 = q^2 - 2k\cdot q = 0$ for $k^2 = m_\mu^2 = 0$ (see eq. (2.1)). This makes the upper part of the diagram in fig. 1, responsible for the creation of muonium, equivalent to the amplitude of the process $\gamma^* + \gamma^* \to M^0$, where γ^*'s are virtual photons. That is why the obtained cross-section for the reaction $e^- + Z \to Z + M^0 + \mu^-$ has turned out to be proportional to $\ln^3(\gamma_{\mathrm{coll}})$.

3 Influence of a finite radius of a nucleus and a distortion of wave functions of coupled leptons

In this section we estimate the influence of a finite radius of the nucleus Z. According to [17], the form factor of the nucleus with a mass number A can be defined by the expansion

$$\frac{1}{Z}\,F_{1Z}(q^2) = 1 - \frac{1}{6}\,r_A^2\,\vec{q}^{\,2} + O(\vec{q}^{\,4})\,, \tag{3.1}$$

where we identify r_A with the radius of a nucleus with mass number A given by [17]

$$r_A = 1.2\,A^{1/3}\,\mathrm{fm} = 6.1\,A^{1/3}\,\mathrm{GeV}^{-1}. \tag{3.2}$$

Due to the finite value of the nuclear radius, the function $I(\vec{p}_1, \vec{p}^{\,\prime}_1)$ changes as follows:

$$\begin{aligned}&\delta I(\vec{p}_1, \vec{p}^{\,\prime}_1) = \\ &-\frac{\pi}{E_1}\,\frac{1}{3}\,r_A^2 E_1^2\left(-\frac{1}{|x\vec{n}^{\,\prime}_1 - \vec{n}_1|} + 3\,\frac{(x - \vec{n}^{\,\prime}_1\cdot\vec{n}_1)^2}{|x\vec{n}^{\,\prime}_1 - \vec{n}_1|^3}\right)\\ &\times(1 - x - |x\vec{n}^{\,\prime}_1 - \vec{n}_1|)^2 \ln\left(\frac{1}{|x\vec{n}^{\,\prime}_1 - \vec{n}_1| - (1-x)}\right).\end{aligned} \tag{3.3}$$

In the region of the integration over t, dominant for the leading term of the expansion of the form factor $F_{1Z}(\vec{q}^{\,2})$ into the powers of $\vec{q}^{\,2}$, the contribution of the finite radius of the nucleus can be summarized as

$$\begin{aligned}\sigma^{(\vec{e}^{\,-}Z)}_{M^0}(E_1) = &\frac{16 Z^2\alpha^7}{\left(1 + \dfrac{1}{6}\dfrac{r_A^2\Lambda^4}{E_1^2}\right)^2}\,\frac{m_e}{m_\mu^3}\\ &\times \ln^3\left(\frac{E_1}{\Lambda}\right)(1 + \cos\Theta_1\cos\Theta^{\prime}_1\,).\end{aligned} \tag{3.4}$$

For the electron (positron) scattering by $^{222}_{86}\mathrm{Rn}$ with the laboratory energy $E_1 = 27.5\,\mathrm{GeV}$ the correction to the cross-section, caused by the finite value of the nucleus radius (3.2), can be made of the order of 4% varying the parameter Λ from $\Lambda \simeq 1\,\mathrm{GeV}$ to $\Lambda \simeq 0.8\,\mathrm{GeV}$ in comparison with the value of the cross-section (2.28) calculated for $E_1 = 27.5\,\mathrm{GeV}$, $\Lambda \simeq 1\,\mathrm{GeV}$ and $r_A = 0$. Hence, in the Weizsäcker-Williams approximation [18–23] without loss of generality we can treat a nucleus Z in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$ as a point-like particle with electric charge Ze.

In the strong Coulomb field caused by a point-like charge Ze for $Z \sim 100$ the wave functions of the initial electron (positron) and the final muon should be distorted. According to [17], at very high energies and in the eiconal approximation these wave functions can be written in the following form:

$$\begin{aligned}&\Psi_{e^-}(\vec{r}_1;\vec{p}_1,\sigma_1)_{\,\mathrm{in}} = u(\vec{p}_1,\sigma_1)\exp\\ &\times\left\{+\,i\vec{p}_1\cdot\vec{r}_1 + i\,\frac{E_1}{|\vec{p}_1|}\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}_1^{\,2} + (z-s)^2}}\right\},\\ &\Psi_{\mu^-}(\vec{r}^{\,\prime}_1;\vec{p}^{\,\prime}_1,\sigma^{\,\prime}_1)_{\,\mathrm{out}} = u(\vec{p}^{\,\prime}_1,\sigma^{\,\prime}_1)\exp\\ &\times\left\{+\,i\vec{p}^{\,\prime}_1\cdot\vec{r}^{\,\prime}_1 - i\,\frac{E^{\,\prime}_1}{|\vec{p}^{\,\prime}_1|}\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}_1^{\,\prime 2} + (z^{\,\prime}+s)^2}}\right\},\end{aligned} \tag{3.5}$$

where $\vec{\rho}_1$ and $\vec{\rho}^{\,\prime}_1$ are components of the radius-vectors $\vec{r}_1$ and $\vec{r}^{\,\prime}_1$ perpendicular to the momentum $\vec{p}_1$ and $\vec{p}^{\,\prime}_1$, respectively.

In the limit $m_e = m_\mu = 0$ the wave functions (3.5) change to

$$\begin{aligned}&\Psi_{e^-}(\vec{r};\vec{p}_1,\sigma_1)_{\,\mathrm{in}} = u(\vec{p}_1,\sigma_1)\exp\\ &\times\left\{+\,i\vec{p}_1\cdot\vec{r} + i\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}^{\,2} + (z-s)^2}}\right\},\\ &\Psi_{\mu^-}(\vec{r};\vec{p}^{\,\prime}_1,\sigma^{\,\prime}_1)_{\,\mathrm{out}} = u(\vec{p}^{\,\prime}_1,\sigma^{\,\prime}_1)\exp\\ &\times\left\{+\,i\vec{p}^{\,\prime}_1\cdot\vec{r} - i\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}^{\,2} + (z+s)^2}}\right\},\end{aligned} \tag{3.6}$$

where we have taken into account the fact that at high energies effectively the production of the final muon occurs at the same spatial point $\vec{r}_1 = \vec{r}^{\,\prime}_1 = \vec{r}$, where the initial electron has been absorbed. The amplitude of the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ is proportional to the product

$$\begin{aligned}&\Psi^\dagger_{\mu^-}(\vec{r};\vec{p}^{\,\prime}_1,\sigma^{\,\prime}_1)_{\mathrm{in}}\Psi_{e^-}(\vec{r};\vec{p}_1,\sigma_1)_{\mathrm{out}} \sim\\ &\exp\left\{i\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}^{\,2}+(z+s)^2}} + i\int_0^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}^{\,2}+(z-s)^2}}\right\} =\\ &\exp\left\{i\int_{-\infty}^\infty \frac{Ze^2\,\mathrm{d}s}{\sqrt{\vec{\rho}^{\,2}+s^2}}\right\} = e^{-i\,Ze^2\,\ln[C\vec{\rho}^{\,2}]}\,,\end{aligned} \tag{3.7}$$

where C is an undefined constant related to the large-distance regularization of the integrals in (3.7). The spinorial factor has been taken already into account for the calculation of the cross-section (2.27) or (3.4).

Formally, the amplitude of the reaction $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ in the momentum representation should be obtained by means of the integration over the configuration space that includes the integration over $\vec{\rho}$ as well. However, due to the presence of the undefined infinitesimal constant C, the integration over $\vec{\rho}$ can be reduced to the replacement of $\vec{\rho}^{\,2}$ by an average value.

Since $|\vec{\rho}\,|$ is a transversal scale of the reaction $\vec{e}^{\,-}+Z \to Z+M^0+\vec{\mu}^{\,-}$, which can be treated as an impact parameter of this reaction, for an estimate of an average value of this parameter we can set $\vec{\rho}^{\,2} \sim \sigma^{(\vec{e}^{\,-}Z)}(E_1)_{\max} \sim \ln^3(E_1/\Lambda)$.

This yields

$$\Psi^{\dagger}_{\mu^-}(\vec{r};\vec{p}\,'_1,\sigma\,'_1)\Psi_{e^-}(\vec{r};\vec{p}_1,\sigma_1) \sim e^{-i\,Ze^2\,\ln[C\,'\ln^3(E_1/\Lambda)]}\,. \tag{3.8}$$

As the cross-section for the reaction is proportional to $|\Psi^{\dagger}_{\mu^-}(\vec{r};\vec{p}\,'_1,\sigma\,'_1)\Psi_{e^-}(\vec{r};\vec{p}_1,\sigma_1)|^2$, the distortion of the wave functions of the initial and final leptons caused by the strong Coulomb field does not change crucially the cross-section for the reaction $\vec{e}^{\,-}+Z \to Z+M^0+\vec{\mu}^{\,-}$ calculated for the wave functions of the coupled leptons in the form of plane waves.

For the estimate of the influence of the strong Coulomb field on the state of muonium we suggest to calculate the time of the decay $M^0 \to \mu^+ + e^-$ induced by the external Coulomb field. The amplitude of the decay $M^0 \to \mu^+ + e^-$, we define as

$$\mathcal{M}(M^0 \to \mu^+ + e^-) = \int \frac{1}{\sqrt{V}}\,e^{-i\vec{p}\cdot\vec{r}}\,U(\vec{r})\,\frac{1}{\sqrt{\pi a^3_{\rm B}}}\,e^{-r/a_{\rm B}}\,{\rm d}^3r, \tag{3.9}$$

where $\vec{p}$ is a relative momentum of the μ^+e^- pair, $a_{\rm B} = 268.173\,{\rm MeV}^{-1}$ is the Bohr radius of muonium, and V is a normalization volume. Then, $U(\vec{r})$ is the potential energy of the dipole moment $\vec{d} = e\,\vec{r}$ of the μ^+e^- pair coupled to the strong Coulomb field of the nucleus Z

$$U(\vec{r}) = -\vec{d}\cdot\vec{E}(\vec{r}) = \frac{Ze^2}{r}\,. \tag{3.10}$$

Integrating over $\vec{r}$ we get

$$\mathcal{M}(M^0 \to \mu^+ + e^-) = \frac{4\pi Ze^2}{\sqrt{V\pi a^3}}\,\frac{a^2_{\rm B}}{1 + a^2_{\rm B}p^2}\,. \tag{3.11}$$

The time of the decay $M^0 \to \mu^+ + e^-$ is equal to

$$\tau^{-1}(M^0 \to \mu^+e^-) = \frac{32Z^2\alpha^2}{a^3_{\rm B}E^2_{M^0}} = \frac{32Z^2\alpha^5}{E^2_{M^0}}\left(\frac{m_e m_\mu}{m_e + m_\mu}\right)^3, \tag{3.12}$$

where E_{M^0} is the energy of the muonium in the rest frame of the nucleus Z. Since $E_{M^0} \gg 5\,{\rm GeV}$, for $^{222}_{86}{\rm Rn}$ we estimate $\tau(M^0 \to \mu^+e^-) \gg 2.6\times 10^{-8}\,{\rm s}$. The time of the interaction of the electron scattering by radon, during which muonium can be produced, is of the order $\tau \sim 10^{-8}\,{\rm s}$. This means that the strong Coulomb field does not affect crucially the production of muonium or anti-muonium in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$. Of course, a more detailed analysis of the Coulomb distortion of the wave functions of leptons in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$ and the influence of this distortion on the production of muonium M^0 and anti-muonium $\bar{M}^0$ is required. We are planning to analyse this problem in our forthcoming investigations.

4 Conclusion

We have calculated the cross-sections for the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$ of the production of muonium M^0 and anti-muonium $\bar{M}^0$ with polarized μ^- and μ^+ mesons by polarized electrons and positrons coupled at high energies to the nucleus Z.

The cross-sections are calculated in dependence on i) the energy E_1 of the initial electron and positron in the laboratory frame, coinciding with the rest frame of a target nucleus Z, and ii) polarizations of the initial electron and positron and final muons in the kinematical region $\omega^2 = (p'_1 + k)^2 \gg m^2_\mu$, making the massless limit of coupled leptons reasonable.

For the numerical estimate of the cross-sections at the energies available for the HERA Collider at DESY [15], *i.e.* $E_1 = 27.5\,{\rm GeV}$, we suggest to use radon, $^{222}_{86}{\rm Rn}$, as target nucleus, since radon has spin 1/2. The theoretical values of the cross-sections for longitudinally polarized electrons and positrons scattering by $^{222}_{86}{\rm Rn}$ are equal to $\sigma^{(\vec{e}^{\,-}{\rm Rn})}_{M^0}(E_1 = 27.5\,{\rm GeV}) = \sigma^{(\vec{e}^{\,+}{\rm Rn})}_{\bar{M}^0}(E_1 = 27.5\,{\rm GeV}) = 1.6\,{\rm pb}$. For these cross-sections we predict the following numbers of favourable events: $N_{M^0} = 808$ and $N_{\bar{M}^0} = 3360$. Hence, the increase of luminosities of electron and positron beams should make the experiment for a test of CPT invariance, suggested by Choban and Kazakov in ref. [12], feasible at DESY.

We have estimated the influence of the finite nuclear radius and the Coulomb distortion of the wave functions of the leptons. According to our estimate in the kinematical region $\omega^2 = (p'_1 + k)^2 \gg m^2_\mu$ the Weizsäcker-Williams approach, treating a nucleus as a point-like particle and neglecting the Coulomb distortion of the wave functions of incoming and outcoming leptons, is a rather well-defined approximation. The contribution of the finite nuclear radius can be kept at the level of a few percents. The distortion of the wave functions of the initial and final leptons caused by the strong Coulomb field does not change the cross-sections for the reactions under consideration. During the time of the production of muonium or anti-muonium, the strong Coulomb field, induced by the charge of the nucleus Ze, does not destroy the bound states of the μ^+e^- or μ^-e^+ pairs. Hence, the strong Coulomb field can hardly screen the phenomenon of the violation of CPT invariance in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$.

We have shown that the test of CPT invariance in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$ reduces to the experimental analysis of the ratio $R(T) = N_{M^0}(T)/N_{\bar{M}^0}(T)$ (1.15) of the numbers of favourable events detected during an interval T. If $R(T)$ is a constant in time —CPT invariance is conserved, and if $R(T)$ is an oscillating function in time, one can conclude that CPT invariance is violated.

We would like to accentuate that this is a qualitative analysis of CPT invariance. In the case of the ratio $R(T)$ oscillating in time we can infer neither a strength nor a nature of violation of CPT invariance.

We argue that the appearance of longitudinally polarized muons in the final states of the reactions $\vec{e}^{\,-} + Z \to Z + X + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{X} + \vec{\mu}^{\,+}$ with longitudinally polarized electrons and positrons is a distinct signal for the production of muonium M^0 and anti-muonium $\bar{M}^0$ with a total spin $J = 0$. This should testify that $X \equiv M^0$ and $\bar{X} \equiv \bar{M}^0$ with a total spin $J = 0$.

Indeed, the creation of the $\mu^+\mu^-$ pairs in the reactions $\vec{e}^{\,\mp} + Z \to Z + \vec{e}^{\,\mp} + \mu^+ + \mu^-$ seems to be the main process competing with the production of muonium and anti-muonium in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$. The main distinction of the production of the $\mu^+\mu^-$ pairs from the production of muonium and anti-muonium is a strong correlation between the momenta and polarizations of μ^+ and μ^- and a decorrelation of them with the initial electron or positron. In turn, a strong correlation between the polarizations of the final muons and the initial electron and positron is a feature of the production of muonium and anti-muonium with a total spin $J = 0$ in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$. Hence, at first glimpse, for the experimental realization of the test of CPT invariance in the reactions $\vec{e}^{\,-} + Z \to Z + M^0 + \vec{\mu}^{\,-}$ and $\vec{e}^{\,+} + Z \to Z + \bar{M}^0 + \vec{\mu}^{\,+}$ with longitudinally polarized electrons and positrons it suffices to count the number of longitudinally polarized μ^- and μ^+ mesons during an interval T. Plotting the ratio of these numbers, which should coincide with $R(T)$, one should obtain an experimental information about CPT invariance.

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